INDE 301 Engineering Economy

Chapter 4 Nominal and Effective Interest Rates

- **Example**
  - You placed \$100 in a saving account for one year at an interest rate of 1% *per month*.
  - Calculate the amount of interest and annual interest rate.
    - $F = P(1+i)^n = 100 \times 1.01^{12} = \$112.68.$ The interest earned is $112.68 - 100 = \$12.68$.
    - The annual interest rate is 12.68/100 = 12.68%.
  - We say that the *effective* annual interest rate is 12.68%.
  - Or, <u>the interest rate is 12% per year, compounded monthly</u>.
  - That is, the effective annual interest that corresponds to a 12% *nominal* annual interest, *compounded* monthly is 12.68%.
- **Nominal interest rate**
  - A nominal interest rate is an interest rate that does not include any consideration of compounding.
  - Means “in name only”, “not the true, effective rate.” E.g.,
  - 12% per year, compounded monthly
    - 12% is NOT the true effective rate (per year)
    - 12% represents the nominal rate
  - Nominal interest rate is commonly referred to as “APR” (annual percentage rate).

- **Effective interest rate**
  - Effective interest rate is the actual rate that applies for a stated period of time.
  - It takes into account the effect compounding of interest
  - Effective interest is stated in the following form:

    r (per year), compounded every CP.
  - It involves two parameters
    - The annual nominal rate r.
    - The compounding period, CP, the time where interest applies
    - E.g.,
      - Daily compounding, CP = 1 day = 1/365 year.
      - Weekly compounding, CP = 1 week =1/52 year.
      - Monthly compounding, CP = 1 month = 1/12 year.
      - Quarterly compounding, CP = 3 months = 1/4 year.
      - Semiannual compounding, CP = 6 months = 1/2 year.
  - The effective rate is called APY (annual percentage yield).

- **Factors under *m*-time-a-year compounding**
  - Under compounding over a period $CP = 1/m$ year (e.g., CP = 1/12 year = 1 month), and at a nominal interest rate r, a present current P is equivalent after k periods (e.g. months) to

$$F = P(1+r/m)^k \Rightarrow (\mathrm{P/F}, r, m, k) = (1+r/m)^k .$$

  - Similarly, F dollars after k periods are equivalent to

$$P = F / (1+r/m)^k \Rightarrow (\mathrm{F/P}, r, m, k) = 1 / (1+r/m)^k .$$

- **Computing the effective interest rate**
  - Note that the effective interest rate per *CP* is r / m, where $m = 1/CP$, with *CP* given in fraction of a year, is the number of times interest is compounded per year.
  - E.g., with monthly compounding, $m = 12$, and a nominal rate of 12% translates into an effective monthly rate of 1% .
  - With semiannual compounding, $m = 2$, and a nominal rate of 12% translates into an effective semiannual rate of 6% .
  - Then, \$1 is equivalent to $(1+r/m)^m$ after 1 year.
  - The effective annual rate is such that $1+i=(1+r/m)^m$. I.e.,

$$i=\left(1+\frac{r}{m}\right)^m-1.$$

- **Continuous compounding**
  - If the compounding period, *CP*, is too small, $CP \rightarrow 0$, the number of compounding times gets too large, $m \rightarrow \infty$.
  - This situation is known as "continuous compounding."
  - Under continuous compounding, the rate of growth of an investment over a time t expressed in years, i.e., the P/F factor, is

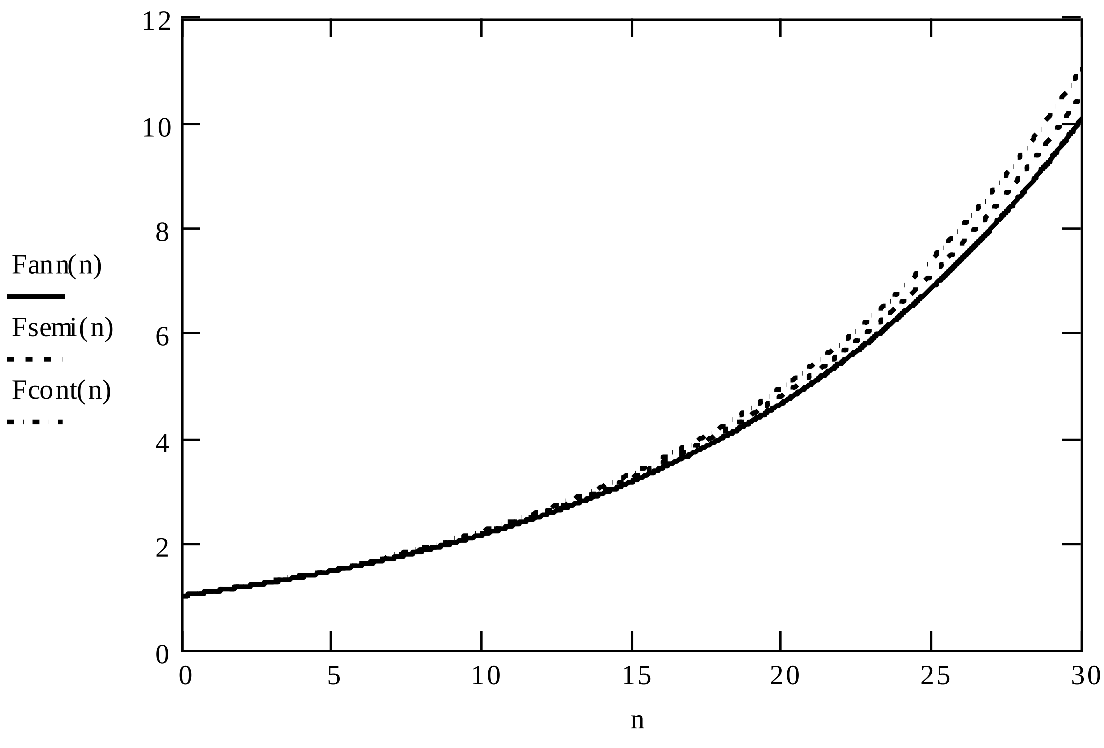
$$\lim_{m\to\infty}(1+r/m)^{mt}=e^{rt}$$

  - Similarly, under continuous compounding, the effective rate is $i=e^r-1$.

- Continuous compounding is often assumed in quantitative finance as it simplifies the analysis.

- **Example: $1 invested at a nominal rate of 8%**

Year	$\boldsymbol{m} = \mathbf{1}$	$\boldsymbol{m} = \mathbf{2}$	$\boldsymbol{m} = \mathbf{4}$	$\boldsymbol{m} = \mathbf{12}$	$\boldsymbol{m} = \boldsymbol{\infty}$
1	1.080	1.082	1.082	1.083	1.083
2	1.166	1.170	1.172	1.173	1.174
3	1.260	1.265	1.268	1.270	1.271
4	1.360	1.369	1.373	1.376	1.377
5	1.469	1.480	1.486	1.490	1.492
6	1.587	1.601	1.608	1.614	1.616
7	1.714	1.732	1.741	1.747	1.751
8	1.851	1.873	1.885	1.892	1.896
9	1.999	2.026	2.040	2.050	2.054
10	2.159	2.191	2.208	2.220	2.226



- **Bonds**
    - Bonds represent the major source that governments and companies use to obtain debt financing.
    - A bond is an obligation by the bond issuer to pay money to the bond holder (buyer).
    - A bond pays its *face value* or *par value* at its maturity date. In addition, bonds usually pay periodic coupon payments. In the U.S., coupon payments are made every 6 months.
    - The coupon amount is described in percent of face value.
    - For example, a 10% coupon with a face value of $1000 will pay $100 coupon per year. If payment is semiannual, the coupon payment will be $50.
    - Usually coupon rates are close to the prevailing interest rate.
    - A bond can be traded freely in the market place. Its price varies continuously.
    - A bond's *yield to maturity* is the interest rate at which the PV of coupon and face value payments are equal to the bond price. This is always quoted on an annual basis.
    - Yield to maturity (YTM) is actually the internal rate of return (IRR) of the bond (to be discussed later).
    - Consider a bond with a price of P and a face value F, making m coupon payments per year of C/m (with a total of n payments).

- the YTM is the value of λ such that

$$P = \frac{F}{(1+\lambda/m)^n} + \sum_{k=1}^{n} \frac{C/m}{[1+(\lambda/m)]^k} \ .$$

- This formula assumes that the interest is compounded every payment period.
- Upon simplification,

$$P = \frac{F}{[1+(\lambda/m)]^n} + \frac{C}{\lambda}\left(1 - \frac{1}{[1+(\lambda/m)]^n}\right) .$$

- This formula implies that the price of the bond is decreasing in its yield. That is, a high-yield bond will have a "low" price (this is the investor's point of view).
- Bond yields are quoted in the financial media.
- E.g., Lebanese treasury bills yield (source: BLOM brief)

Treasury Yields

	19/12/08	05/12/08	Change bps
3-M TB yield	5.16%	5.18%	-2
6-M TB yield	7.12%	7.17%	-5
12-M TB yield	7.64%	7.66%	-2
24-M TB coupon	8.34%	8.38%	-4
36-M TB coupon	9.02	9.04%	-2
60-M TB coupon	11.50%	11.50%	0

- **Bond example**
  - What is the price of a 10% (coupon, paid semiannually), 30-year US Treasury bond with yield 4%? Assume a face value of 100. (Bond prices are typically quoted as percentage of face value.)
  - In this example, F = \$100, C = 0.1×100=\$10, and λ = 4%, n = 30× 2= 60.

$$P = \frac{F}{[1+(\lambda/m)]^n} + \frac{C}{\lambda}\left(1 - \frac{1}{[1+(\lambda/m)]^n}\right)$$

$$= \frac{100}{(1+0.04/2)^{60}} + \frac{10}{0.04}\left(1 - \frac{1}{(1+0.04/2)^{60}}\right) = \$204.28$$

- **Interest rate that varies with time**
  - In practice, interest rate may vary from one period to the other.
  - In particular, it is often *expected* that the interest rate will *increase* with time.
  - This fact is reflected in the annual yield on long (maturity) bonds being higher than that on short bonds. (See Lebanese TB above.)
  - If the interest rates in periods, 1, …, n are i_1, …, i_n. Then, the future worth after n periods, F, of a present amount P is

$$F = P(1+i_1)(1+i_2)\ldots(1+i_n) = P\prod_{t=1}^{n}(1+i_t)\,.$$

- The rates $i_1, \ldots, i_n$ are known as *short rates*.
- The short rate i_t represent the expected 1-year rate after t years.
- Short rates are estimated based on the yields of bonds similar to those above.